Hydrodynamics of elastic micro-filaments : model comparison and applications

Clément Moreau, Laetitia Giraldi, Hermes Gadêlha

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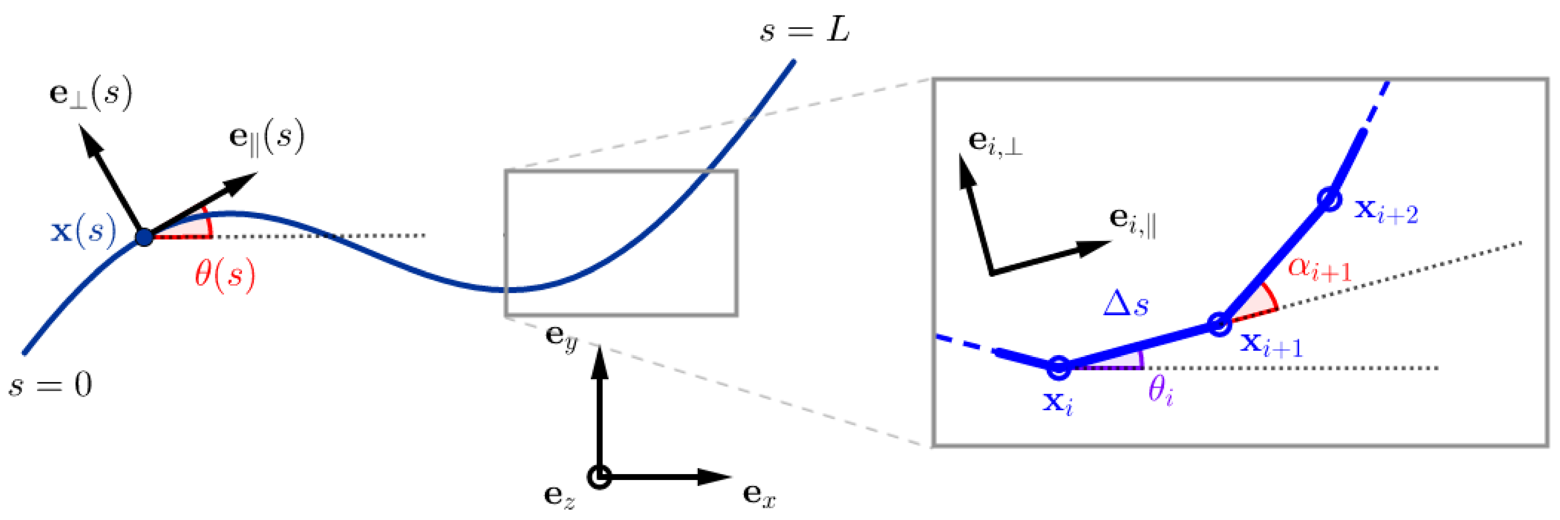
Motivations

- Efficiently model and simulate the dynamics of **inextensible micro-filaments** in 2D
- Deal with the coupling between elastic and hydrodynamic interactions
- Propose a model that is
  - **simple** to implement
  - numerically **robust**
  - **adaptable** for a range of applications

Modeling

- We consider an **inextensible flexible filament** immersed in a fluid at **low Reynolds number** → inertia is negligible, viscous effects dominate: **Stokes regime**



Parametrisation for the continuous and coarse-grained models

- **Hydrodynamics**: segment force density (Resistive Force Theory [2])

$$\mathbf{f}_i(s) = \eta_\parallel(\mathbf{v}_i(s)\cdot\mathbf{e}_{i,\parallel})\mathbf{e}_{i,\parallel} + \eta_\perp(\mathbf{v}_i(s)\cdot\mathbf{e}_{i,\perp})\mathbf{e}_{i,\perp}.$$

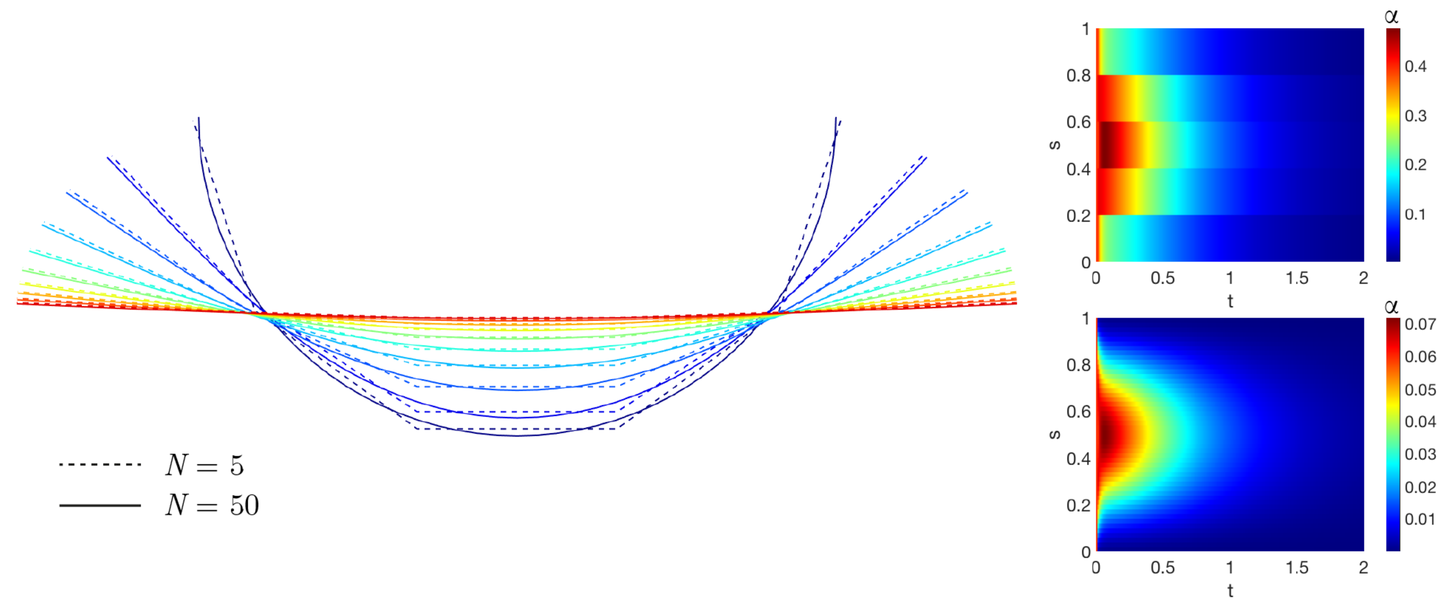
- **Elasticity**: linear model

$$\mathbf{M}_i^{el} = \kappa(\theta_{i+1} - \theta_i)\mathbf{e}_z$$

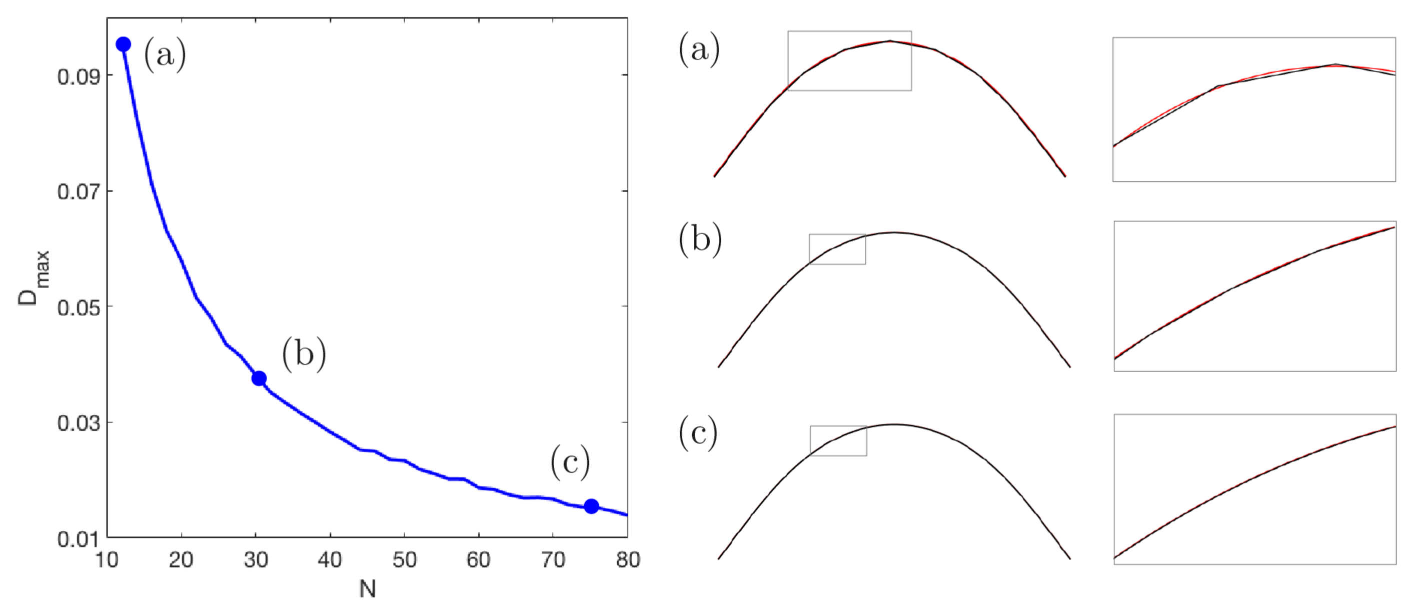
- 2nd Newton law gives the balance of forces and torques :
  - Continuous model → **PDE system**
  - Discrete model → integration on the segments → **ODE system**

Numerical Comparison

- Similar behavior on the "relaxation test"



- Fast convergence of the discrete model (error criterion $D_{max} = \max_{s,t} |x_c - x_{cg}|$)

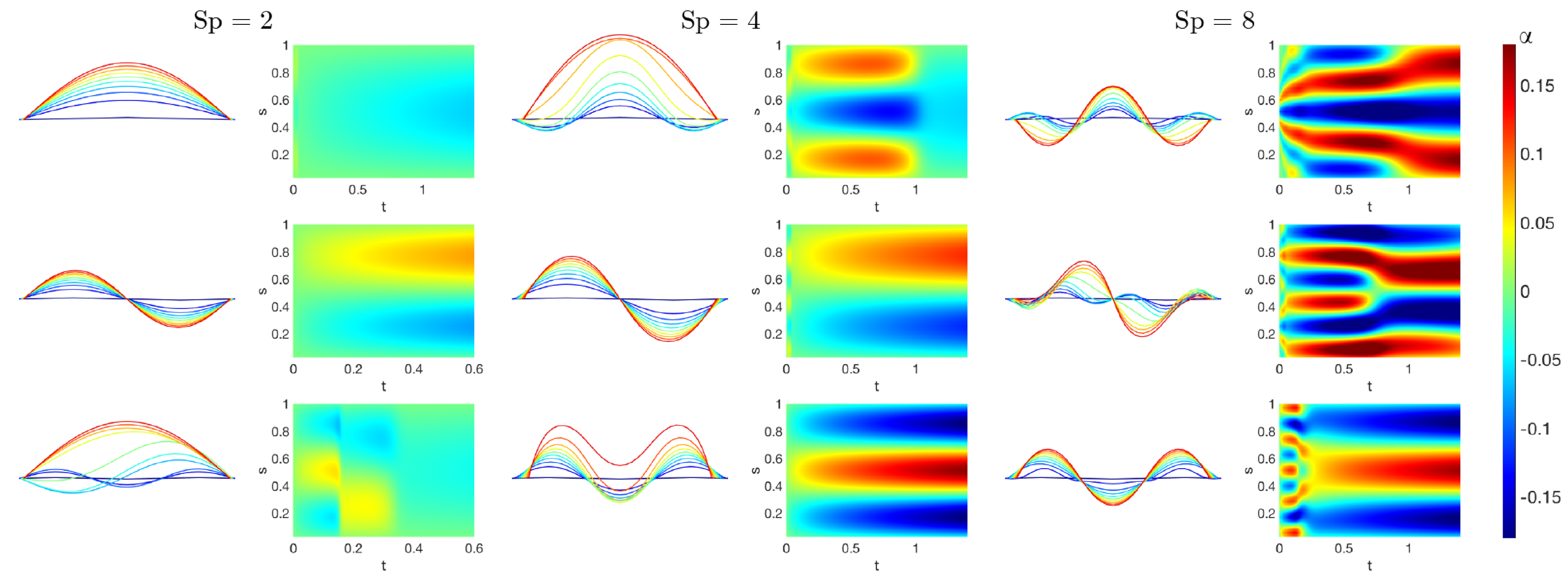


Comparison between the classical and coarse-grained systems for increasing number of segments.

- Discrete approach up to **100 times faster** than PDE approach
- **More robust** for rigid filaments and sharp curvatures

Applications : Buckling

- Filament submitted to **compression**
- Triggering different modes by changing initial condition



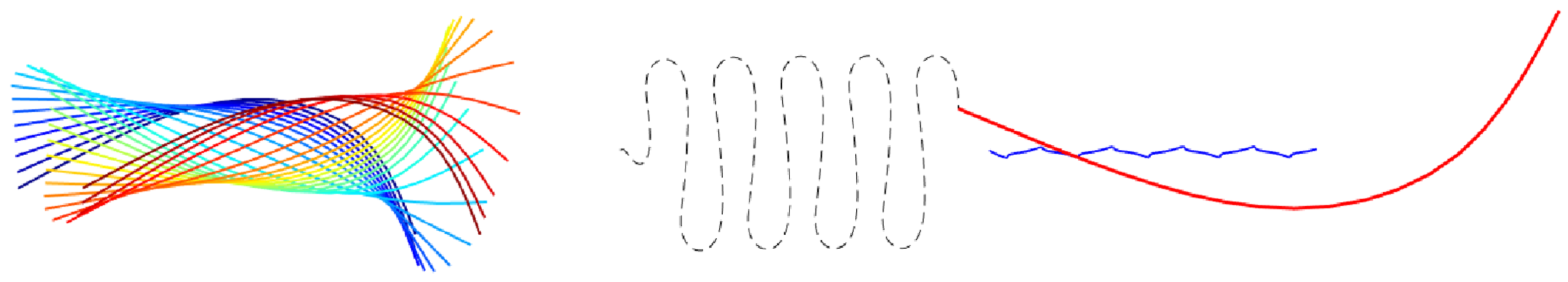
Visualization of the buckling phenomenon for three different rigidities and three different initial conditions.

- Study made easier by the discrete approach, under investigation

Applications : Magnetic Swimmers

- **Magnetised filament** – magnetisations μ_i
- External magnetic field **H** creating a torque

$$\mathbf{T}_i^m = \mu_i(\mathbf{e}_{i,\parallel} \times \mathbf{H})$$

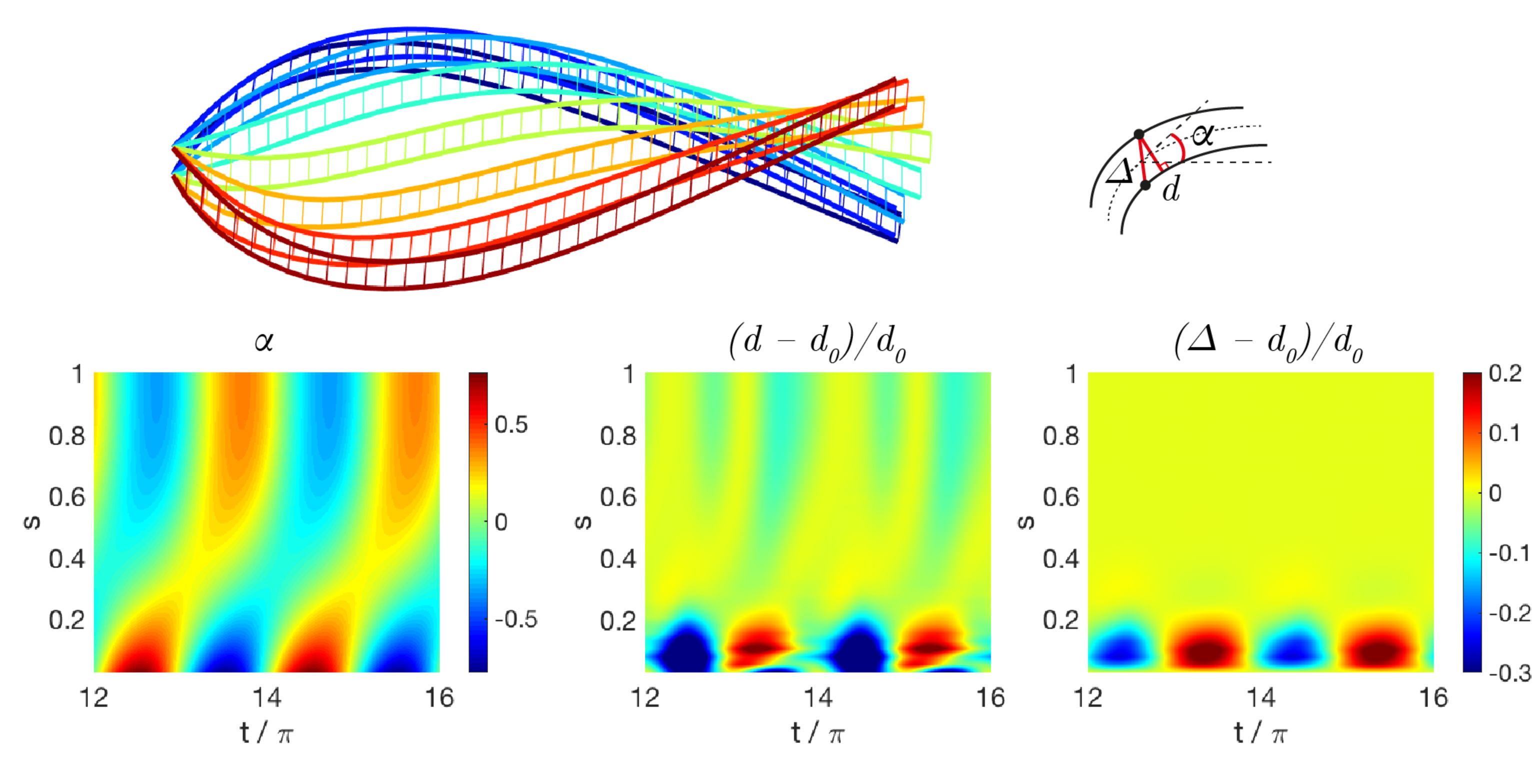


Example of magnetic drive with a sinusoidal orthogonal magnetic field.

- **Artificial self-propulsion** at micro-scale [3]
- Controllability and **optimal control**

Applications : Filament Bundle

- Model a **biological flagellum** through two filaments linked together
- Highlight non-trivial coupling phenomena



Coupling between two filaments obtained with the coarse-grained approach.

- Applications include sperm cell motility studies

References and Contact Information

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